



## **Development of new dynamic elastic constant estimation method for FRP and its validation using FDTD method**

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### **Abstract**

FRP (Fiber Reinforced Plastic) is known as one of the anisotropic materials and has been used as a member of aircraft, civil and architectural structures. The anisotropic property sometimes makes it difficult for inspectors to detect flaws in FRP because the phase and group velocities of the ultrasonic waves in anisotropic materials depend on their propagation directions. These velocities of the ultrasonic waves in a material can be determined by using the dynamic elastic constants of the material. Therefore, it is important task to determine the dynamic elastic constants of the anisotropic material such as FRP. In this research, a new dynamic elastic constant estimation method using the laser ultrasonic visualization testing is developed for FRP. The laser ultrasonic visualization technique is incorporated to measure the ultrasonic wave velocities which are required for elastic constant estimation. In this paper, the procedure of our developed elastic constant estimation method is described with the anisotropic elastodynamic theory. An image processing technique for the visualized data obtained by the laser ultrasonic visualization testing is utilized to determine the phase and group velocities of the ultrasonic waves. The elastic constants of an FRP specimen are estimated by our proposed method. In addition, the simulation of ultrasonic wave propagation in the FRP with the estimated elastic constants is implemented by using the finite difference time-domain (FDTD) method and the results are compared with those obtained by the laser ultrasonic visualization testing to validate our proposed method.

### **1. Introduction**

Some anisotropic materials have been attracted lots of interest in the fields of the mechanical and civil engineering in recent years. The austenitic stainless steel and FRP are known as typical anisotropic materials. In fact, the CFRP and GFRP, which are kinds of FRP, are widely used as the main materials of the latest aircraft and automobile. In general, the ultrasonic non-destructive testing is known as one of the most popular inspection method for the material structures. However, the anisotropic property sometimes makes it difficult for the inspectors to detect a defect in material structures because the phase and group velocities of the ultrasonic waves depend on the propagation direction (1) due to the anisotropic property. Therefore, the inspectors must understand the anisotropic property of the materials to be inspected for accurate inspection. In general, the phase and group velocities of the ultrasonic waves can be calculated by using the elastic constants of the material to be inspected. Therefore, the

development of the dynamic elastic constant estimation method is considered as very valuable in the field of the ultrasonic non-destructive evaluation. Several researches on the elastic constant estimation of a material with anisotropic property have been done since several decades (2)(3). However, the critical disadvantages of conventional elastic constant estimation methods are that the material to be inspected must be cut and immersed in the water to examine the phase and group velocities of the ultrasonic waves propagating obliquely in the material. Therefore, in this research, a new elastic constant estimation method using a laser ultrasonic visualization testing is developed for FRP. A laser ultrasonic visualization technique which permits non-contacting inspection is utilized to obtain the phase and group velocities of the ultrasonic waves. In what follows, the fundamentals of anisotropic elastodynamic theory are presented. Next, the procedure of the new elastic constant estimation method using a laser ultrasonic visualization testing is described. As an example using the proposed method, the elastic constants of a CFRP (Carbon FRP) test specimen with transversely isotropic property are estimated. The finite difference time-domain (FDTD) simulation of the ultrasonic waves propagating in the CFRP is implemented and the visualized wave fields obtained by the FDTD are compared with those by the corresponding laser ultrasonic visualization testing to validate the proposed method.

## 2. Fundamentals of anisotropic elastodynamic theory

In this section, the fundamentals of anisotropic elastodynamic theory are discussed. A Latin suffix takes the values 1, 2 and 3, unless otherwise stated. In addition, the summation convention is valid for repeated indices throughout this paper. The equation of motion and constitutive equation are written as follows:

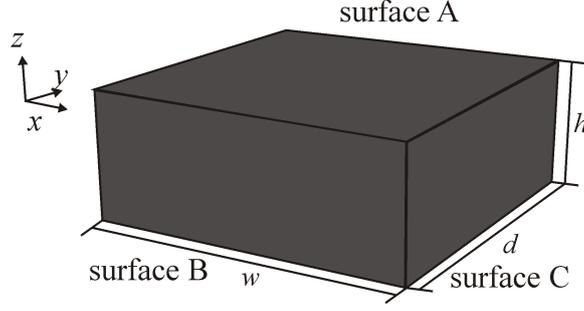
$$\rho \ddot{u}_i(\mathbf{x}, t) = \sigma_{ij,j}(\mathbf{x}, t) \quad (1)$$

$$\sigma_{ij}(\mathbf{x}, t) = C_{ijkl} u_{k,l}(\mathbf{x}, t) \quad (2)$$

where  $\rho$  is the density,  $u_i(\mathbf{x}, t)$  is the displacement at the position  $\mathbf{x}$  and the time  $t$ ,  $C_{ijkl}$  is the elastic constant. In addition,  $\sigma_{ij}$  is the stress tensor,  $(\dot{\quad})$  and  $(\quad)_{,i}$  are the time and spatial derivative, respectively. In Eq.(1), the body force term is omitted. The fourth order elastic constant  $C_{ijkl}$  can be transformed into  $C_{\alpha\beta}$  ( $\alpha, \beta = 1, \dots, 6$ ) expressed in the Voigt notation as follows:

$$\alpha = \begin{cases} i & : i = j \\ 9 - (i + j) & : i \neq j \end{cases}, \quad \beta = \begin{cases} k & : k = l \\ 9 - (k + l) & : k \neq l \end{cases} \quad (3)$$

Equation (3) has the following associations: if  $i, j = 1, 2$  or  $3$ , the stress components in Eq.(2) correspond to axial components. On the other hand, if  $i, j = 4, 5$  or  $6$ , they correspond to the shear components. The objective of this research is to develop a new method to estimate the elastic constant  $C_{\alpha\beta}$  of a FRP. The dynamic elastic constant  $C_{\alpha\beta}$  is related to the phase and group velocities of the ultrasonic waves. Substituting a plane wave equation with the polarization vector  $\mathbf{d}$ , propagation vector  $\mathbf{n}$  which are unit vectors, and the phase velocity  $c$  into Eq. (1), the Christoffel equation can be obtained as follows (4):



**Figure 1. A model for the test specimen and the coordinate system.**

$$(\Gamma_{ik} - \lambda \delta_{ik})d_k = 0, \Gamma_{ik} = C_{ijkl}n_j n_l, \lambda = \rho c^2 \quad (4)$$

where  $\Gamma_{ik}$  is the Christoffel tensor. Equation (4) shows the characteristic equation with the eigenvalues  $\lambda$  and eigenvectors  $\mathbf{d}$ . Three distinct eigenvalues  $\lambda$  and eigenvectors  $\mathbf{d}$  can be obtained by solving the Christoffel equation. The corresponding three phase velocities  $\mathbf{c}$  can be calculated from Eq.(4) once three eigenvalues  $\lambda$  and eigenvectors  $\mathbf{d}$  are obtained. It can be seen from this fact that different three waves, two shear waves (qS1 and qS2 waves) and one longitudinal wave (qP wave) exist in anisotropic materials.

### 3. Relation between elastic constant and phase velocity

Let us consider a CFRP with transversely isotropic property as shown in Fig.1. It is assumed that the shape of a CFRP is rectangular parallelepiped. The elastic constant  $C_{\alpha\beta}$  can be calculated using the phase velocities in anisotropic materials as follows (5):

$$C_{11} = \rho V_{L-L}^2 \quad (5)$$

$$C_{22} = \rho V_{L-C}^2 \quad (6)$$

$$C_{33} = \rho V_{L-Z}^2 \quad (7)$$

$$C_{44} = \rho V_{T\perp LZL-Z}^2 \quad (8)$$

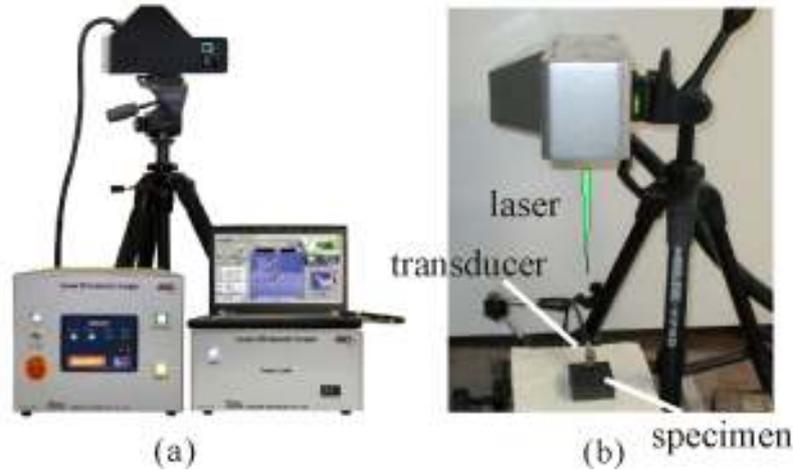
$$C_{55} = \rho V_{T\perp LZC-Z}^2 \quad (9)$$

$$C_{66} = \rho V_{T\perp LZL-L}^2 \quad (10)$$

$$C_{23} + C_{44} = \frac{1}{2} \sqrt{(4\rho V_{L-ZC}^2 - C_{22} - C_{33} - 2C_{44})^2 - (C_{22} - C_{33})^2} \quad (11)$$

$$C_{13} + C_{55} = \frac{1}{2} \sqrt{(4\rho V_{L-ZL}^2 - C_{11} - C_{33} - 2C_{55})^2 - (C_{11} - C_{33})^2} \quad (12)$$

$$C_{12} + C_{66} = \frac{1}{2} \sqrt{(4\rho V_{L-LC}^2 - C_{11} - C_{22} - 2C_{66})^2 - (C_{11} - C_{22})^2} \quad (13)$$



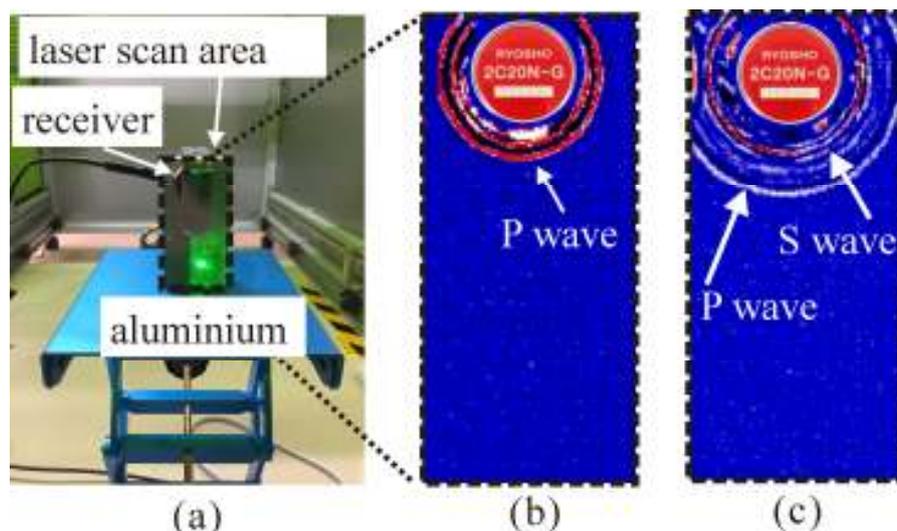
**Figure 2. Laser ultrasonic visualization testing (a) laser ultrasonic visualization testing system (b) excitation of laser ultrasonic waves.**

where  $V_{L-L}$ ,  $V_{L-C}$  and  $V_{L-Z}$  are the phase velocities of qP waves traveling to the  $x$ ,  $y$  and  $z$  direction, respectively. In addition,  $V_{T\perp ZL-Z}$ ,  $V_{T\perp ZC-Z}$  and  $V_{T\perp ZL-L}$  are the phase velocities of qS waves polarized in the  $y$ ,  $x$  and  $y$  direction propagating to the  $z$ ,  $z$  and  $x$  direction, respectively. In addition,  $V_{L-ZC}$ ,  $V_{L-ZL}$  and  $V_{L-LC}$  are the phase velocities of the qP waves which propagate with an angle of  $45^\circ$  in the  $y-z$ ,  $x-z$ ,  $x-y$  plane, respectively. In Eqs.(5)-(13), the density  $\rho$  is known in advance. Therefore, the elastic constant  $C_{\alpha\beta}$  can be calculated by using Eqs.(5)-(13) if all the phase velocities are obtained.

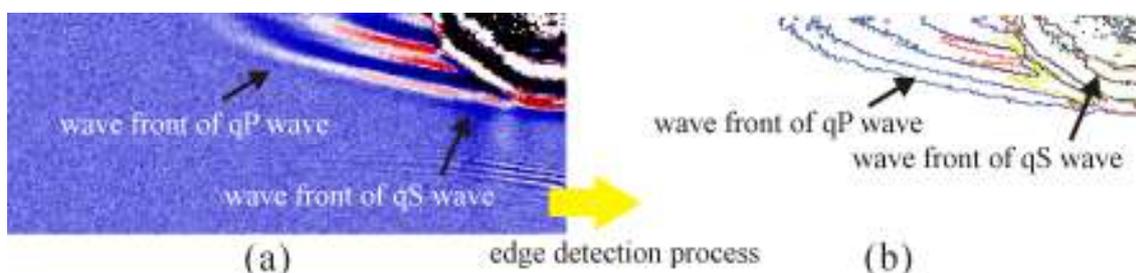
#### 4. Laser ultrasonic visualization testing

The phase velocity is equal to the group velocity for the isotropic materials. Therefore, the phase velocity (or group velocity) in the isotropic materials can be obtained by using standard ultrasonic contacting testing without any difficulties. However, the phase velocity is not consistent with the group velocity for the anisotropic materials. This fact makes it difficult to calculate the elastic constant  $C_{\alpha\beta}$  of the anisotropic materials. In addition, the phase velocities of the qP waves which propagate with an angle of  $45^\circ$  in the  $y-z$ ,  $x-z$ , and  $x-y$  planes are required. These phase velocities cannot be obtained by using standard ultrasonic contacting testing without cutting of the test specimen due to the anisotropic property. Therefore, in this research, a laser ultrasonic visualization testing is utilized to obtain the phase and group velocities.

Figure 2(a) shows an example of the system of the laser ultrasonic visualization testing. The laser ultrasonic visualization testing is one of the non-contacting ultrasonic methods. The laser is used to generate the ultrasonic waves in a specimen. The excited ultrasonic waves are received by the contacting transducer on the specimen as shown in Fig.2(b). The state of the ultrasonic wave propagation on the surface of the specimen can be visualized by using this system. Figure 3 shows an example of the experimental result for an aluminum specimen obtained by the laser ultrasonic visualization testing. As shown in Fig.3(b) and (c), the P and S wave propagation on the front surface of the aluminum specimen in Fig.3(a) can be confirmed visually. The aluminum is known as an isotropic material. Therefore, it can be observed that the P and S waves propagate



**Figure 3. Laser ultrasonic testing for an aluminum specimen (a) experiment setup (b) visualized ultrasonic wave propagation at  $t=2.8\mu s$  and (c)  $t=4.2\mu s$ .**

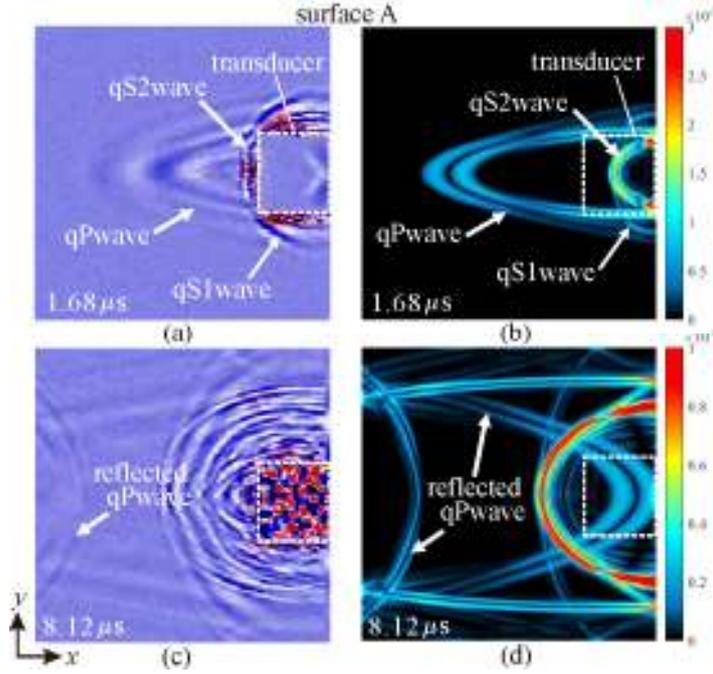


**Figure 4. Edge detection process (a) an original snapshot (b) edge processed snapshot.**

isotropically with a unique speed in the aluminum specimen. Therefore, as shown in Fig.3, it is possible to visually confirm the wave propagating in the direction of  $45^\circ$  obliquely in the surface of test specimen without cutting it by using the laser ultrasonic visualization testing.

## **5. Elastic constant estimation by using the laser ultrasonic testing**

In this section, the brief description of the new elastic constant estimation method is presented. As mentioned in the previous section, the visualized ultrasonic wave fields on the surfaces of a specimen can be obtained by using the laser ultrasonic visualization testing. However, unfortunately, the wave fronts in the visualized results obtained by using the laser ultrasonic visualization testing as shown in Fig.3 are not always clear due to some mechanical noises. Therefore, in this research, the edge detection method, which is one of the image processing techniques, is applied to the visualized data obtained by the laser ultrasonic visualization testing in order to clarify the wave fronts. Figure 4 (a) and (b) show a visualized data obtained by the laser ultrasonic visualization testing for the unidirectional CFRP, whose details are given in the next section, and the corresponding edge detection processed result, respectively. As you can see, the wave fronts seen in Fig.4(a) can be transformed into those in Fig. 4(b) using the edge



**Figure 5. Results for the laser ultrasonic visualization testing and FDTD simulation. (a) and (c) show the ultrasonic wave propagation at surface A in Fig.1. (b) and (d) are the corresponding results obtained by using FDTD.**

detection technique. The group velocities can be calculated by the time difference between two any edge-detected pictures. In addition, the group velocity of the qP wave propagating in  $45^\circ$  to the horizontal axis can be easily obtained by the same way using two any edge-detected pictures. All phase velocities appeared in Eqs.(5)-(13) can be obtained by these operations for all surfaces of the test specimen. Once all phase velocities are obtained, the elastic constants  $C_{\alpha\beta}$  can be calculated by using Eqs.(5)-(13).

## 6. Elastic constant estimation results

### 6.1 Estimation results

In this section, the elastic constants of the unidirectional CFRP with the density  $\rho = 1600\text{kg/m}^3$ , as shown in Fig.1, are estimated by the proposed method. The width  $w$ , depth  $d$ , and height  $h$  of the unidirectional CFRP are  $w = 5\text{cm}$ ,  $d = 5\text{cm}$  and  $h = 2\text{cm}$ , respectively. The 45 degree angle probe with the central frequency  $f = 1\text{MHz}$  is used as the ultrasonic receiver and the 500 KHz high-pass filter is used. The matrix notation of the estimated elastic constants for this CFRP can be seen as follows:

$$C_{\alpha\beta} = \begin{pmatrix} 144.95 & 3.87 & 3.87 & 0 & 0 & 0 \\ & 14.19 & 6.03 & 0 & 0 & 0 \\ & & 14.19 & 0 & 0 & 0 \\ & & & 4.08 & 0 & 0 \\ \text{sym.} & & & & 6.09 & 0 \\ & & & & & 6.09 \end{pmatrix} \text{GPa} \quad (14)$$

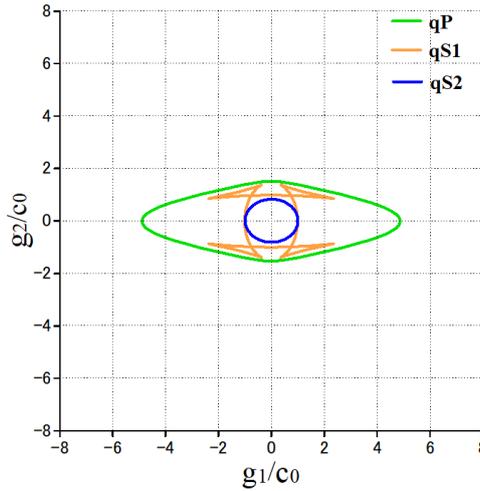


Figure 6. Group velocity curves of the test specimen.

## 6.2 FDTD simulation using the estimated elastic constants

Next, the FDTD method (6) is carried out for the ultrasonic wave propagation problems to verify the proposed method. There are some numerical methods for wave analyses for anisotropic materials (7). However, in this research, the FDTD is selected as simulation tool because of simplicity and sufficiency to represent the problem. The detail of the FDTD is omitted due to limitations of space. The estimated elastic constant  $C_{\alpha\beta}$  is directly used for the input material parameters in this simulation. In general, it is difficult to give the same incident ultrasonic wave for the FDTD simulation as the one used in the laser ultrasonic visualization testing. Therefore, in this analysis, the following time-domain incident ultrasonic wave with central period  $T(= 2\pi/\omega)$  and angular frequency  $\omega = 2\pi f$  is given as the point force:

$$u_3(\mathbf{x}, t) = \begin{cases} \sin\left(\frac{2\pi}{T}t\right) & \text{for } 0 \leq t \leq T \\ 0 & \text{for otherwise} \end{cases} \quad (15)$$

This incident ultrasonic wave  $\dot{u}_3(\mathbf{x}, t)$ , which is the particle velocity component for the  $x_3$  direction, is given as the boundary condition. In addition, the traction free boundary condition is imposed on the three surface of the CFRP as shown in Fig.1.

Figure 6 shows the results for the laser ultrasonic visualization testing and for its corresponding FDTD simulation. Figures 5(a) and (c) demonstrate the ultrasonic wave propagation in the surface A of the unidirectional CFRP in Fig.1, and Figs. 5(b) and (d) are the corresponding results obtained by the FDTD simulation. The total wave fields  $|\mathbf{u}|$  are shown in Figs.5(b) and (d) to compare with those by the laser ultrasonic visualization testing. As seen in Figs.5(a), the qP, which is faster than qS, and qS waves are observed. The velocity of the qP wave for the horizontal direction is faster than that for the vertical direction due to the anisotropic property of the unidirectional CFRP. In addition, the reflected qP wave can be confirmed in Fig.5(c). The same tendency as the laser ultrasonic visualization testing results can be seen in those obtained by FDTD

simulation in Figs.5(b) and (d). The group velocity curves of this CFRP test specimen are shown in Fig.6. These group velocity curves are calculated by using Eq.(14). The wave fronts of the qP and qS waves seen in Fig.5 concord with the group velocity curves in Fig.6. Therefore, it is concluded that the estimation of the unidirectional CFRP with transversely isotropic property is generally successful.

## 7. Conclusions

In this paper, the new elastic constant estimation method using the laser ultrasonic visualization testing was presented. The present method was implemented for a unidirectional CFRP specimen with transversely isotropic property. The ultrasonic wave fields calculated by the FDTD method using the estimated elastic constants of the specimen were compared with the measured ones in the experiment to validate the proposed method. The other approach to estimate the elastic constants of a unidirectional CFRP can be seen in the paper (8). In the future, this method will be applied to the elastic constant estimation of various laminated CFRPs.

## Acknowledgements

This work is supported by JSPS KAKENHI Grant Number 17H0329400.

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