A comparison of three approaches for the inversion of ultrasonic Rayleigh wave data into residual stress profiles

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Abstract

In various industries the surfaces of highly-loaded materials and components are specially treated to increase their resistivity to wear, corrosion and stresses. By shot-peening, laser shock-peening, heat-treatment or other methods residual stresses are deliberately generated. In order to characterize these in the near-surface areas, ultrasonic (Rayleigh) surface waves can be applied. In this contribution, we study the possibility of retrieving the near-surface residual stress state of a material from synthetic Rayleigh wave data. Several formulations that have been proposed in the past in various contexts are compared. It is shown that this particular problem may reveal in relevant materials undesired behaviors for some methods that could be reliably applied to infer other properties. The instabilities of a method based on a Taylor expansion are explained by highlighting singularities in the series of coefficients. At the same time, it is shown that a method based on a piecewise linear expansion regularized through a Singular Value Decomposition (SVD) can successfully provide performances which only weakly depend on the material. Orders of magnitudes are given for residual stress profiles generated by shot-peening. Systematic errors and sensitivities to noise are compared on a limited frequency range, with a focus on lower and intermediate frequencies.

1. Introduction

This communication is mostly based on Ref. (1). We report on the forward problem via perturbation theory to generate the Rayleigh wave dispersion data in dependence of an input stress profile. Several formulations for the inverse procedure are introduced. Finally, we present our evaluation results. The methods detailed in this work can be applied to real cases where other sources of dispersion are negligible. Applications related to heat-treated metals (2) or tempered glass (3) have already successfully been pointed at.

2. Forward problem via perturbation theory

The change in Rayleigh wave velocity $v_R$ produced by a stress profile $\sigma (z)$ at angular frequency $\omega = k v_R$ can be expressed, under a hypothesis of small residual strains, as:

$$\frac{\Delta v_R}{v_R} (\omega) = \int_0^\infty \sigma (z) f_i e^{-k z n_i k} dz,$$

in which $f_i$ and $n_i$ are constants that depend on the second and third order elastic properties of the material (see Ref. (1)).
3. Formulations of the inverse procedure

Several ways of inverting Eq. (1) are examined: a first one based on a Taylor expansion of the profile and a polynomial fit of the frequency data, and a second one based on a piecewise linear expansion regularized through a SVD. Performances of other methods based on strong shape assumptions are also briefly commented on.

![Figure 1. (from Ref. (1), Fig. 6) Reconstruction obtained with the piecewise linear expansion and SVD regularization. The input synthetic dispersion data is simulated in [2 MHz - 15 MHz] assuming a profile inspired from X-ray measurements on shot-peened titanium samples (empty/full squares for parallel/perpendicular stress component). 10% Gaussian noise is added.]

4. Conclusions

This study warns that inverse methods which only rely on shape assumptions are likely to be ill-behaved when applied to residual stresses, in comparison to other properties. It also shows how a material-dependent regularization step can solve the issue.

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References