



Experimental and theoretical investigation of edge waves propagation and scattering in a thick plate with surface-breaking crack-like defect

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Abstract

The results of theoretical and experimental investigations of edge waves (EW) and Lamb guided waves (LW) excitation, propagation and diffraction in isotropic thick plates with a surface-breaking crack-like defect are reported in the present paper. In the experimental studies, being conducted for plate-like aluminium samples, EW and LW excitation is achieved by thin piezoelectric transducers adhered to a surface or an edge of the specimen. Laser Doppler vibrometry is employed to measure velocities on the surface of the plate. A computational model employed for the evaluation of EW dispersion properties and source-induced transient EW and LW time-space wave-fields in a thick plate is based on the three-dimensional linear elastodynamic theory for a half-bounded plate and the normal mode expansion technique. The present study focuses on the investigation of guided waves scattering by surface-breaking cracks of different depth at an edge. It is demonstrated that the intensity of EW reflection is steadily increasing with frequency.

1. Introduction

Elastic wave phenomena in thin-walled structures are widely used for the purposes of non-destructive evaluation. Along with the Lamb guided waves (LW), which have been widely studied and adopted in many engineering areas, edge waves (EW) propagating along a stress-free edge of a thin elastic plate are of particular interest in non-destructive testing. They have potential applications in the measurement of material properties and crack detection and identification in many safety-critical structures, e.g., propellers or turbine blades, in which defects initiate at the specimen edges.

Theoretical studies demonstrated that a number of edge waves can propagate in thick elastic plates (see [1-5] and review [6]). The authors showed recently that proposed mathematical models predict properties of EW with a good accuracy, see [7] for more details. In this study, which is based on theoretical results [6], experimental investigations of EW and LW diffraction by a surface-breaking crack-like defect in isotropic thick plates are presented. Proposed computational model [7] for the evaluation of EW dispersion properties and source-induced transient EW and LW time-space wave-fields in a thick plate is based on 3D linear elastodynamic theory for a half-

bounded plate and employs the normal mode expansion technique. In the experimental studies, being conducted for plate-like aluminium samples, EW and LW excitation is achieved by thin piezoelectric wafer active sensor (PWAS) adhered to a surface or to an edge of the specimen. Laser Doppler vibrometry is employed to approve the developed mathematical models experimentally. The calculated dispersion curves are in a good agreement with results of wavenumber-frequency analysis of measured signals, and a reliable correspondence between simulated and measured transient signals is demonstrated. The present study also focuses on the investigation of EW scattering by surface-breaking cracks of different depth at an edge of a thick elastic plate. Influence of depth and central frequency on wave transmission and scattering in the damaged zone is considered.

2. Experimental setup

To investigate wave scattering by crack-like defects of various depths a number of experimental studies have been performed. The Cartesian coordinates are introduced as shown in Figure 1. The depth of the defect is denoted as d , its value in the experiments is 0.0 mm, 0.5 mm, 1.0 mm, 1.9 mm, 3.0 mm. Aluminium plate sample with dimensions 600 mm x 400 mm x 4.85 mm has been used in the experiments. Guided wave packets are generated in the plate by a rectangular PWAS with dimensions 30 mm x 5 mm x 0.25 mm. The PWAS manufactured by PI Ceramic GmbH is adhesively attached to the edge of the specimen.

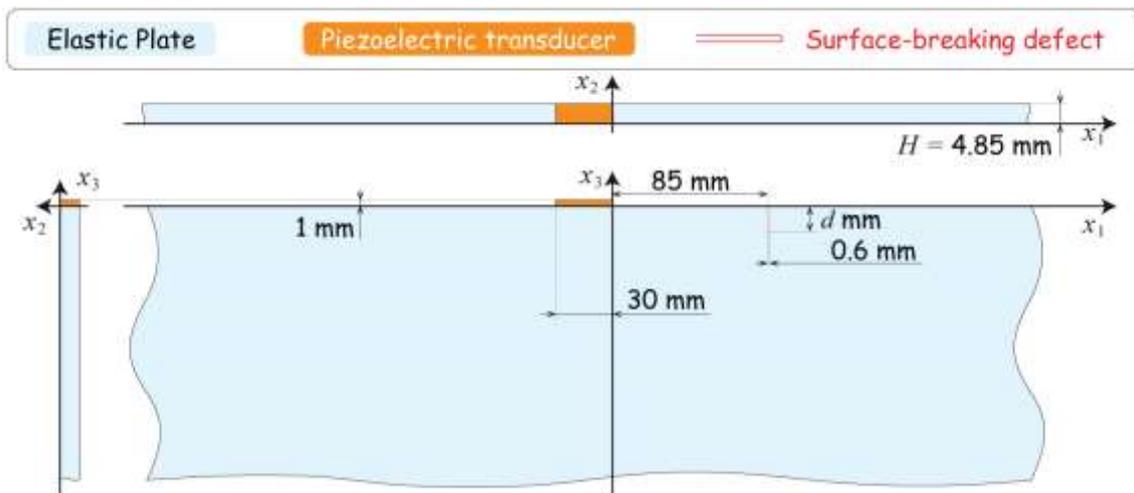


Figure 1. Geometry of the problem

The actuator is driven with a transient voltage in the form of either a broad-band rectangular pulse of short duration or a narrow-band Hann-windowed five-cycle sine burst, which spectrum is concentrated near its central frequency f_0 . The latter are generated by a Tektronix AFG 3022B arbitrary signal generator and are pre-amplified to the range 70 V-pp by a NF HSA4101 external high-frequency power amplifier before being applied to the PWAS. The out-of-plane velocity field of propagating waves is measured on the plate surfaces by means of a Polytec PSV-500 one-dimensional scanning laser Doppler vibrometer. The photographs of the experimental set-up and damaged part of the specimen are depicted in Figure 2.

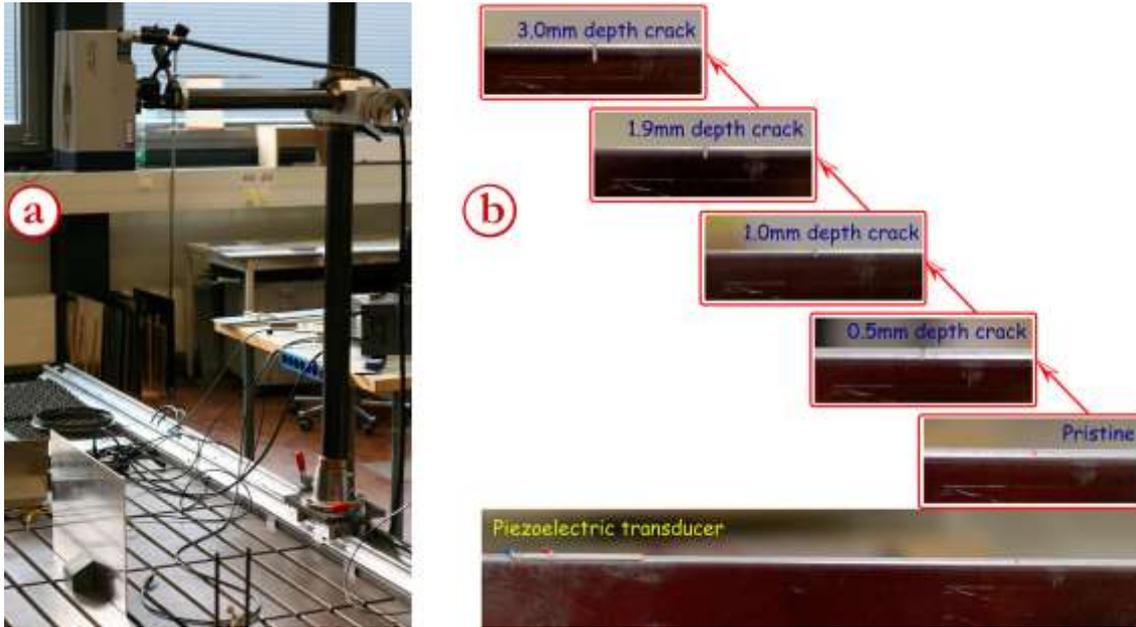


Figure 2. Photo of experimental setup and specimen: (a) Laser Doppler vibrometer situated above the specimen; (b) defects of different depths at the edge of the specimen

3. Analysis of wave-field scattered by defects

The pristine plate with PWAS was used in the first set of experiments. The velocities excited by the Hann-window signal with central frequencies $f_0 = 100, 300, 500, 700$ kHz applied at the PWAS were measured. Two regions were chosen for the measurements: part of the edge $\Omega_e = \{0.5 < x_1 < 250, x_3 = 0\}$, and $\Omega_e = \{55 < x_1 < 115, 0.5 < x_3 < 20.5, x_2 = 0\}$ at the face of the thick plate. After 0.5 mm depth cut was made by a saw, measurements of elastic waves excited by the PWAS and scattered by the defect were made again. The same procedure was repeated for enlarged defect of depths 1.0 mm, 1.9 mm and 3.0 mm.

First, the waves propagating along the edge of the plate are considered. The out-of-plane velocities were measured along the middle line of the edge, i.e. $x_2 = H/2, x_3 = 0$. The same measurements were made at central frequencies 100, 300, 500, 700 kHz for pristine thick plate and cuts of four different depths located 85 mm from the nearest PWAS edge. The Hilbert transform $H[u_3]$ over time variable t has been applied to the acquired out-of-plane velocities u_3 as demonstrated in Figures 3 and 4. The Hilbert transform gives more condensed information about amplitudes of the wave motion. One can see that the defect does not influence on wave propagation along the edge at $f_0=100$ kHz and reflected waves are almost indistinguishable at the surfaces in the left part of Figure 3.

At the central frequency $f_0=300$ kHz, the effect of the surface-breaking defect is visible even if the depth of crack is 0.5 mm. In the case of 3.0 mm depth defect, the amplitudes of waves transmitted through the damaged zone are reduced more than four times. The

defect of 1.9 mm depth does not reflect so much ES0 fundamental mode as 3.0 mm depth defect, though reflected waves are clearly visible in Figure 3.

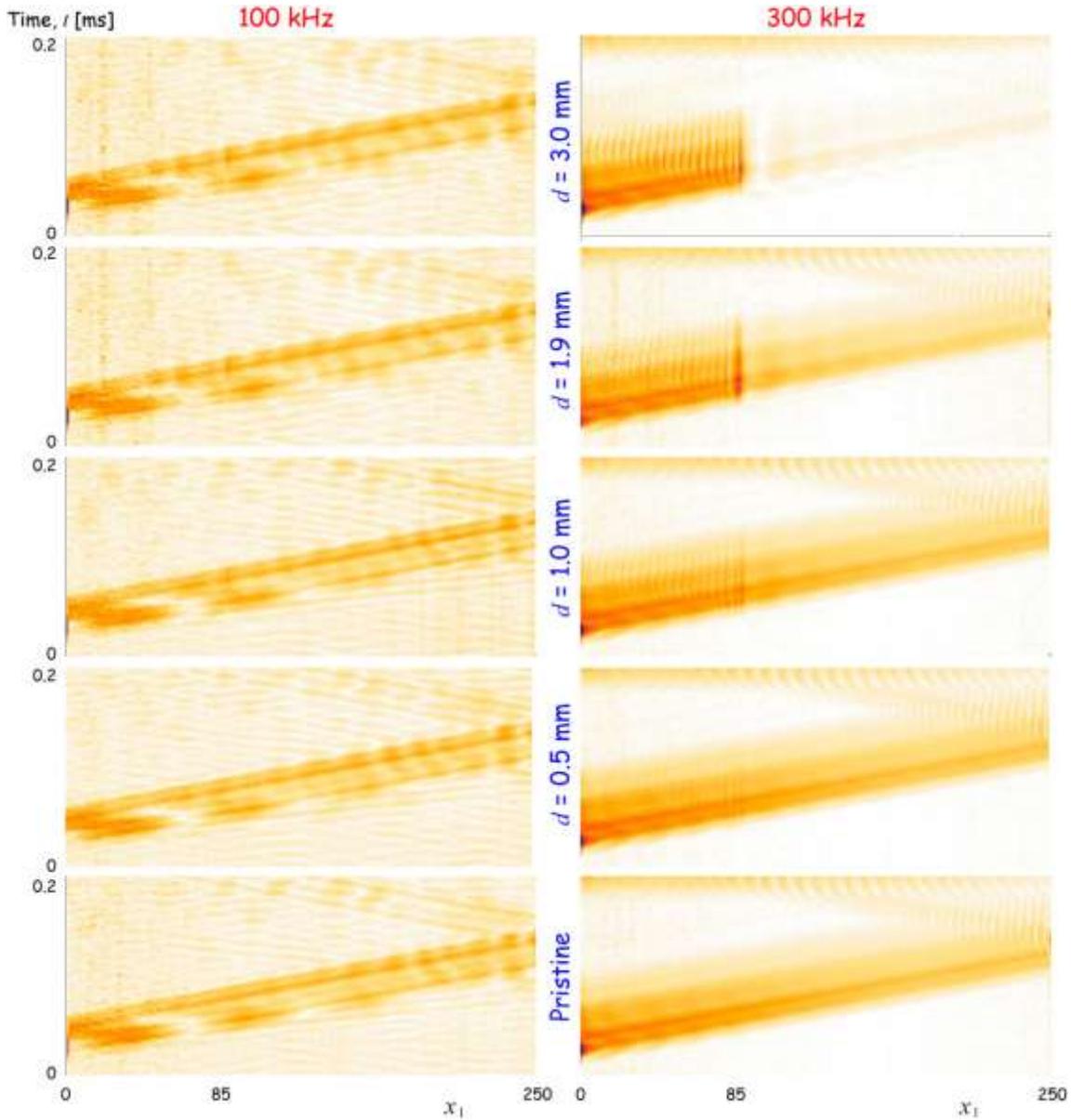


Figure 3. Hilbert-enveloped out-of-plane velocities $H[\dot{u}_3(x_1, H/2, 0)]$ measured on the edge of the plate at central frequencies 100 kHz and 300 kHz

In [7], where the authors studied mechanism of guided waves excitation by the PWAS at the edge, it was demonstrated that fundamental mode ES0 is predominant at the edge. Mode ES0 is not localized in the vicinity of the edge surface at lower frequencies. Amplitudes of ES0 decay faster with depth at higher frequencies. Variation of three components u_i of the displacement vector of ES0 mode along vertical coordinate x_3 are presented in Figure 5. It is clearly seen that vibration is more concentrated in the vicinity of the edge at higher frequencies. Accordingly, low reflection by defects at central frequency $f_0=100$ kHz can be explained by these properties of ES0 mode. This fundamental EW carries the main part of its energy outside the crack if $f_0=100$ kHz.

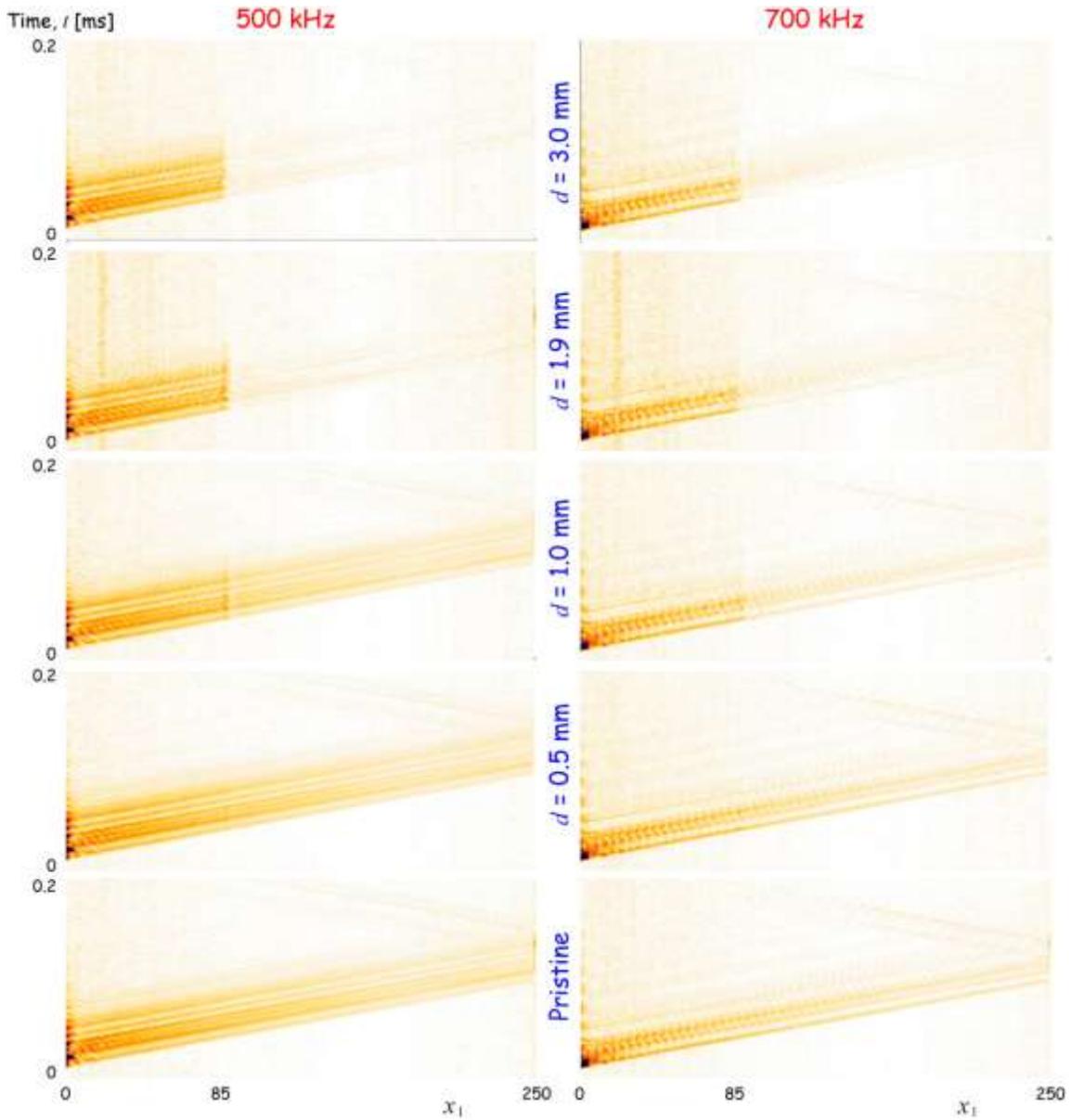


Figure 4. Hilbert-enveloped out-of-plane velocities $H[\dot{u}_3(x_1, H/2, 0, t)]$ measured on the edge of the plate at central frequencies 500 kHz and 700 kHz

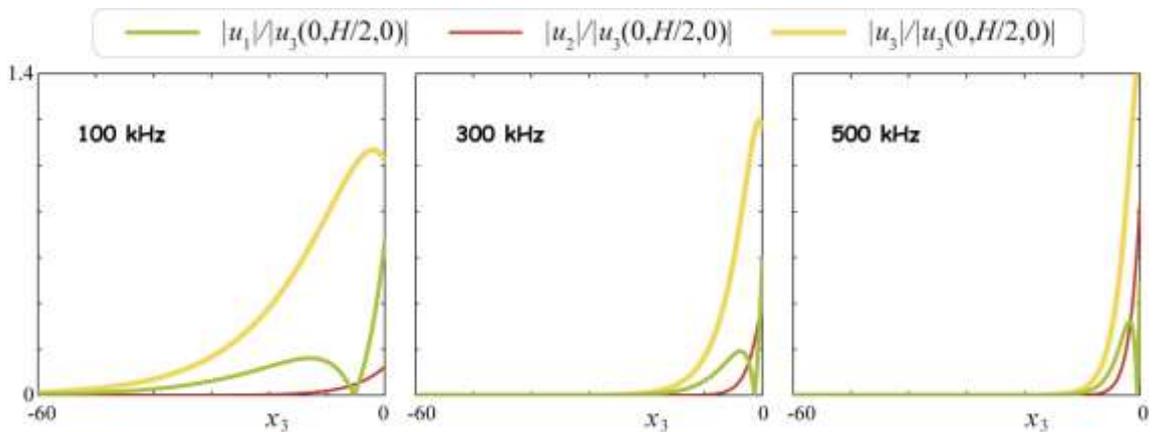


Figure 5. Displacements u_i of fundamental edge wave ES0 at frequencies 100, 300, 500 kHz

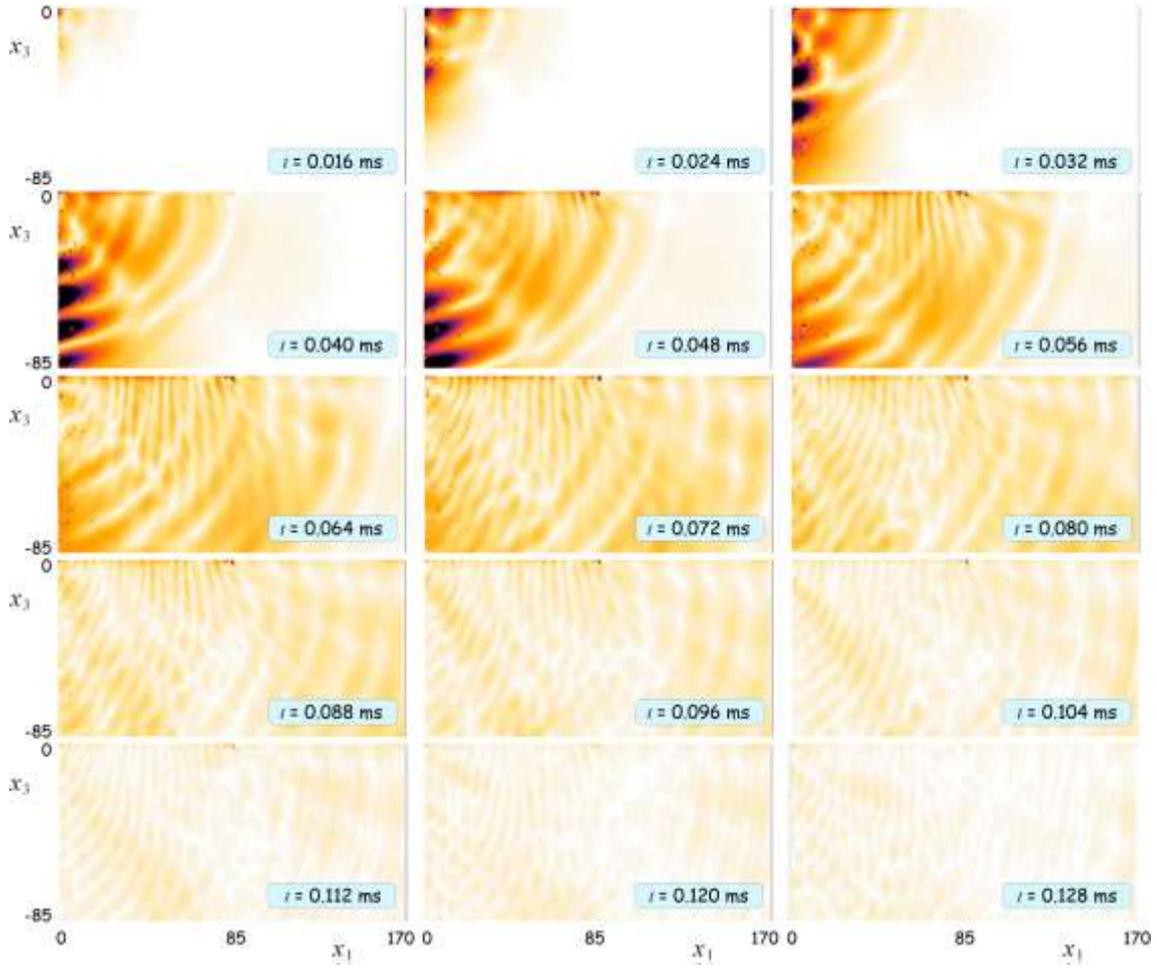


Figure 6. Hilbert-enveloped out-of-plane velocities $H[\dot{u}_2(x_1, 0, x_3)]$ measured on the surface of the plate with defect of depth $d = 3$ mm at central frequency 300 kHz at different moments of time t

At the central frequencies $f_0=500$ kHz and $f_0=700$ kHz the amplitudes of EW decay fast on the crack length when the latter is 1 mm or above. This result of the theoretical investigations fully agrees with the experimental results presented in Figure 4. Moreover, the studies of higher order EW in [5] allow to explain the strong excitation of vibration near the PWAS that can be observed in Figure 4. Here we see wave ES1 excited near its cut-off frequency where the damping because of link with Lamb waves is relatively strong.

In order to investigate more carefully wave phenomena accompanying scattering by a surface-breaking edge crack out-of-plane velocities were also measured at the face of the specimen. The central frequency $f_0=300$ kHz was chosen for the specimen with 3.0 mm depth crack due to the intensive scattering of incoming edge waves. Figure 6 shows snapshots of the Hilbert-enveloped out-of-plane velocities $H[\dot{u}_2(x_1, 0, x_3)]$ measured on the face of specimen. The defect is located in the centre of the scanned area. It can be seen that the crack mainly reflects ESO mode backward and reradiates it into Lamb waves. Accordingly, transmitted ESO waves are of much lower amplitudes compared to the reflected ESO waves. Further experimental study shows that some

resonance phenomenon occurs at 315 kHz. Theoretical calculations showed that width of plate is approximately half wavelength of the corresponding wedge wave at 290 kHz.

5. Conclusions

The measurements of out-of-plane velocities excited by the PWAS at the edge of 4.85 mm thickness aluminium plate with an artificial crack made as a cut were provided using Laser Doppler vibrometry. A low reflection by the crack at lower frequencies (below central frequency $f_0=150$ kHz) is observed. At higher frequencies, the surface-breaking defect is distinguishable due to reflected EW and LW. At the central frequency 315 kHz, a resonance scattering related to a wedge wave resonance is revealed. Such reflection properties and resonance are explained theoretically by the properties of edge wave ES0 and wedge wave.

Acknowledgements

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