Numerical modeling of Nonlinear Coda Wave Interferometry in a multiple scattering medium with a localized micro-cracked zone

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Abstract

The spectral element method is used to perform a parametric sensitivity study of the Nonlinear Coda Wave Interferometry (NCWI) method in a heterogeneous sample with a localized micro-cracked zone (1). The influence of a strong pump wave on localized nonlinear damage has been modeled as modifications of the elastic properties in an Effective Damage Zone (EDZ), depending on the pump wave amplitude. To match the previous experimental results of Zhang \textit{et al.} (2), the corresponding change of the attenuation coefficient has to increase of 700\%, which is unexpectedly high. To mitigate this effect and to be closer to real defects, random cracks of random orientations are added to the EDZ. The results demonstrate that the variation of elastic properties required to match the experimental results are reduced. The numerical results reported constitute another step towards quantification and forecasting of the nonlinear acoustic response of a cracked material, which proves to be necessary for quantitative non-destructive evaluation.

1. Introduction

In the field of evaluation and Non-Destructive Testing (NDT) on complex materials in civil engineering, to obtain a better spatial resolution and to improve the sensitivity, the general idea of the conventional ultrasonic methods is to increase the frequency, but in the meantime the attenuation is increased, which is mainly due to intrinsic absorption and scattering (3-5). The latter reduces the amplitudes of the direct waves and induces the coda wave, which arrives later (6,7). The coda wave has been shown to be reproducible (8) which contains abundant information of the medium, and it has a very high sensitivity to disturbances of the propagation medium (9-11). Nonlinear acoustic methods are a sort of ultrasonic methods which are studied a lot for the complex materials due to their high sensitivity to nonlinear diffusers (12-14). Among the various nonlinear acoustic methods, nonlinear modulation is widely studied to characterize and detect damage to materials (15,16). The idea is to apply waves of high amplitude, but low frequency (pump wave), and then to probe the material with a high frequency probe wave, the purpose of the pump wave is to highlight the nonlinear effects of the damaged material. Based on this, a new set of methods, combining the Nonlinear modulation method and the Coda Wave Interferometry (NCWI) is proposed by Zhang \textit{et al.} in 2013 for a sample of glass (2) and applied to the mortar by Hilloulin \textit{et al.} (11). In this context, we have performed numerical parametric studies (1), using the spectral elements method (SEM) (17), to reproduce coda wave signal modifications induced by...
a pump wave of varying amplitude as in the experience of Zhang et al. (2), with the same material, i.e. glass sample. For each pump amplitude the effective damaged zone (EDZ) in the numerical models is modified by changes its intrinsic properties (Young's modulus and attenuation coefficient) to match the experimental results of Zhang et al. (2). A pump amplitude reaches 60 dB, the corresponding change of the attenuation coefficient has to increase of 700%, which is unexpectedly high. To mitigate this effect and to be closer to real defects, random cracks of random orientations are added to the EDZ and the new numerical model is studied in this paper.

The Coda Wave Interferometry (CWI) method consists of comparing two signals coming from two material states, including a reference state and a damaged state. The two observables to estimate the differences between these two signals are: the relative variation of velocity $\theta$ and the remnant decorrelation coefficient $K_d$. A recent and powerful tool to estimate the change in propagation velocity is called Stretching. It evaluates the correlation coefficient between a perturbed signal $u_p(t)$ and a reference signal $u(t)$, at an expansion rate $\theta_k$ ($k = 1, 2, \ldots, n$ for different levels of expansion) that simulates a global increase/decrease in propagation velocity within the medium (18).

$$\text{CC}(\theta_k) = \frac{\int_{t_c-T}^{t_c+T} u_r(t|1+\theta_k) u_p(t) \, dt}{\sqrt{\int_{t_c-T}^{t_c+T} u_r^2(t|1+\theta_k) \, dt \int_{t_c-T}^{t_c+T} u_p^2(t) \, dt}}$$

The value of the correlation coefficient $\text{CC}(\theta_k)$ in Eq. (1) represents, quantitatively, the similarity of the two signals recorded before and after perturbation of the medium within a selected time window $[t_c-T, t_c+T]$, where $t_c$ is the central time of the window and $2T$ the length of the analyzed window. Herein, two CWI observables are traditionally introduced: i) when $\text{CC}(\theta_k)$ reaches its maximum value, the relative variation in effective coda velocity is introduced and simply denoted as $\theta = \theta_k = \delta v/v$; and ii) in order to quantify the level of distortion between the two signals, the remnant decorrelation coefficient $K_d$ is introduced as $K_d = 100 \left| 1 - \text{CC}(\theta) \right|$.

2. Numerical Configurations

In order to mitigate the effect of the very important change in attenuation coefficient of the previous numerical results and to be closer to real defects, random cracks of random orientations are added to the effective damaged zone (EDZ) (Fig. 1). Each crack is modeled by an empty mesh cell, the crack thickness is $10^{-2}$ mm, therefore negligible compare to the minimum wavelength 4.2 mm. The purpose of putting a void in the cracks is to maximize the contrast along the trajectory of wave propagation so that the waves are completely reflected by the cracks, which is the case in reality for an open crack.
The numerical modeling is made using the two-dimensional spectral element method (SEM2D) (17) and the meshes are realized by the GMSH software. The numerical configuration is shown in Fig. 1, the matrix is always set to $200 \times 200 \text{mm}^2$, the center of the EDZ is located at (155 mm, 140 mm). The emission source is a chirp signal with the frequency band of [200 kHz, 800 kHz] and its position at (50 mm, 200 mm). The duration of the source is 0.2 ms. The matrix outside of the EDZ is homogeneous in this study. Non-linearities are strongly related to damage caused by micro-cracks in different types of materials, not only in nonlinear mesoscopic elastic (NME) materials, but also in damaged homogeneous materials (19, 20).

The characteristics of the medium outside the EDZ (called the matrix material), for all numerical models, are assigned the following properties: the matrix (glass) has a Young’s modulus $E_{\text{mtx}} = 69 \text{Gpa}$, a Poisson’s ratio $\nu_{\text{mtx}} = 0.25$ and a volumetric mass density $\rho_{\text{mtx}} = 2500 \text{kg} \cdot \text{m}^{-3}$, yielding the velocities for P-wave and S-wave of $5755 \text{m} \cdot \text{s}^{-1}$ and $3323 \text{m} \cdot \text{s}^{-1}$, respectively. The CWI observables are relative values resulting from the comparison of two states of the sample. In this study, the reference model corresponds to the case where the material properties are equal in both the matrix and the EDZ to that of intact glass. The perturbed state contains the damaged glass in the EDZ, which is modeled by changes in both Young’s modulus $E_{\text{edz}} = E_{\text{mtx}} \left(1 + \Delta E_{\text{edz}} / E_{\text{mtx}}\right)$ and effective attenuation coefficient $Q_{\text{edz}}^{-1} = Q_{\text{mtx}}^{-1} \left(1 + \Delta Q_{\text{edz}} / Q_{\text{mtx}}^{-1}\right)$. The quality factors for $Q_\lambda^{-1}$ and $Q_\mu^{-1}$ are respectively 1250 and 350 [1]. The matrix outside the EDZ remains unchanged as intact glass. The minimum wavelength is estimated as $\lambda_{\text{min}} \approx 4 \text{mm}$. Consequently, by considering the precision and cost of numerical calculations, the cell size in this study is set at 6 mm.

![Figure 1. Numerical configuration of the following tests: An homogeneous model with a Effective Damage Zones (EDZ) completed by random cracks of random orientations. Each crack is modeled by one void cell, and the size of the crack is measured 10 mm $\times$ 0.01 mm. The blue lines represent the contour of the matrix, the EDZ and the cracks. The matrix size equals 200 $\times$ 200 mm$^2$, and the circular EDZ center is located at (155 mm, 140 mm). The source position is (50 mm, 200 mm). (a) Configuration without mesh; (b) Zoom of the meshed EDZ; (c) Zoom of one crack with the red marks in (b).](image)

3. Numerical results
In this study, the influence of the pump amplitude on the EDZ is modeled by a change in Young's modulus \( \Delta E_{edz}/E_{mtx} \) from -0.80% to 0.00%, as well as a change in the attenuation coefficient \( \Delta Q_{edz}^{-1}/Q_{mtx}^{-1} \) from 0 to 6. The model of which \( \Delta E_{edz}/E_{mtx} = 0, \Delta Q_{edz}^{-1}/Q_{mtx}^{-1} = 0 \) corresponds to the reference model. Only the negative change of the Young's modulus is taken into account by recognizing that softening is typically induced by the pump wave in the previous experiment (1, 2).

In Fig. 2, the changes of the relative variation of velocity \( \theta \) and the remnant decorrelation coefficient \( Kd \) in terms of \( \Delta E_{edz}/E_{mtx} \) and \( \Delta Q_{edz}^{-1}/Q_{mtx}^{-1} \) show different behaviors, which corresponds well to previous numerical studies with no crack in the EDZ [1]. \( \theta \) is more influenced by the variation of the Young’s modulus in the EDZ \( \Delta E_{edz}/E_{mtx} \) and \( Kd \) is mainly influenced by the variation in attenuation \( \Delta Q_{edz}^{-1}/Q_{mtx}^{-1} \), at least within the parameter variation ranges being explored herein. These observations confirm once again the assumptions made in Zhang et al. (2) for the interpretations of experimental observations based on the quadratic hysteresis model.

By using the model with a micro-cracked EDZ, the comparisons between the numerical results and the experimental results of Zhang et al. (2) are restored. At each amplitude level of the pump wave in the experiment, a single group of values of \( \theta \) and \( Kd \) is obtained. The following normalized weighted average \( \overline{O}_{CW} \) is used to compare the estimated experimental values \( (\theta_{exp}, Kd_{exp}) \) of Zhang et al. (2) with the numerical results \( (\theta, Kd) \) obtained from the imposed values of \( \Delta E_{edz}/E_{mtx} \) and \( \Delta Q_{edz}^{-1}/Q_{mtx}^{-1} \) (Fig. 3).
\[ O_{\text{CWI}}^- = \frac{1}{2} \left( 1 - \frac{|\theta - \theta_{\text{exp}}|}{\theta_{\text{exp}}} \right) + \left( 1 - \frac{|Kd - Kd_{\text{exp}}|}{Kd_{\text{exp}}} \right) \]  

(2)

Figure 3. Normalized weighted average of the CWI observables values: the relative variations in velocity \( \theta \) and the remnant decorrelation coefficient \( Kd \). The coefficients of weighted average for \( \theta \) and \( Kd \) are calculated from the group of values \( \theta_{\text{exp}} \) and \( Kd_{\text{exp}} \), which have been extracted from the experimental results of Zhang et al. [2] for a given pump wave amplitude: a) \( A_{\text{pump}} = 40 \text{ dB} \) corresponds to \( (\theta_{\text{exp}} \sim 0.0004, Kd_{\text{exp}} \sim 0.2) \); b) \( A_{\text{pump}} = 50 \text{ dB} \) corresponds to \( (\theta_{\text{exp}} \sim 0.0023, Kd_{\text{exp}} \sim 1) \); c) \( A_{\text{pump}} = 60 \text{ dB} \) corresponds to \( (\theta_{\text{exp}} \sim 0.009, Kd_{\text{exp}} \sim 7) \). The maximum value of the weighted average of CWI observables refers to a group of values \( (\Delta E_{\text{edz}}/E_{\text{mix}}, \Delta Q_{\text{edz}}/Q_{\text{mix}}^{-1}) \), which in turn provide an approximation of the estimated value of CWI observables \( (\theta_{\text{exp}}, Kd_{\text{exp}}) \), i.e. each group of values \( (\Delta E_{\text{edz}}/E_{\text{mix}}, \Delta Q_{\text{edz}}/Q_{\text{mix}}^{-1}) \) results in a maximum weighted average corresponding to a pump wave amplitude level encountered during the experiment.
Figure 4. (a) Property variations in the Effective Damage Zone (EDZ) \( \Delta E_{\text{edz}}/E_{\text{mix}}, \Delta Q_{\text{edz}}^{-1}/Q_{\text{mix}}^{-1} \) compared to the experimental results of Zhang et al. [2] in the two cases of numerical modelings: (Blue) Previous results of the homogeneous EDZ model; (Green) Numerical results with the microcracked EDZ model. top) Change in Young's modulus \( \Delta E_{\text{edz}}/E_{\text{mix}} \) vs. step number; middle) change in effective attenuation coefficient \( \Delta Q_{\text{edz}}^{-1}/Q_{\text{mix}}^{-1} \) vs. step number; and bottom) Pump wave excitation amplitude at each step of the test. (b) Property variations in the Effective Damage Zone (EDZ) \( \Delta E_{\text{edz}}/E_{\text{mix}}, \Delta Q_{\text{edz}}^{-1}/Q_{\text{mix}}^{-1} \) vs. pump wave amplitude in the experiment [2]: top) Change in Young's modulus \( \Delta E_{\text{edz}}/E_{\text{mix}} \) vs. normalized pump amplitude; and bottom) change in effective attenuation coefficient \( \Delta Q_{\text{edz}}^{-1}/Q_{\text{mix}}^{-1} \) vs. normalized pump amplitude. Linear fits of the numerical results are proposed.

The aim of this part is to compare the numerical results in the following two cases: the model with an homogeneous EDZ (1) and the model with a micro-cracked EDZ. The Fig. 4 show the variations of the intrinsic properties in the EDZ \( \Delta E_{\text{edz}}/E_{\text{mix}}, \Delta Q_{\text{edz}}^{-1}/Q_{\text{mix}}^{-1} \) in comparison with the experimental results of Zhang et al. (2) following the two modeling cases: (Blue) Previous results for the model with an homogeneous EDZ; (Green) Numerical model results with a micro-cracked EDZ. As shown in Fig.4, for an given amplitude level of the pump wave \( A_{\text{pump}} \), necessary changes in Young's modulus \( \Delta E_{\text{edz}}/E_{\text{mix}} \) and the coefficient of attenuation \( \Delta Q_{\text{edz}}^{-1}/Q_{\text{mix}}^{-1} \) in the EDZ are actually much lower in the case where the EDZ is filled with the random cracks. For example, by adding 20 random cracks of random orientations in the EDZ, for \( A_{\text{pump}} \) of 60 dB, the required change in the effective coefficient attenuation corresponding to the experimental results decreases about 57% compared to the case of which the EDZ is homogeneous. Multiple scattering by random cracks has clearly a significant effect on the CWI observables. Thanks to the random cracks, the multi-trajectory waves pass more frequently in the EDZ, due to multiple reflections by the crack interfaces, which induces additional attenuation so that the
necessary change of the attenuation coefficient corresponds to the experimental results is effectively reduced.

4. Conclusions

In this paper we have studied the method Nonlinear Coda Wave Interferometry (NCWI) numerically with a new model where cracks of random orientations are added to the Effective Damaged Zone (EDZ). The observables of CWI: the relative variation of velocity $\theta$ and the remnant decorrelation coefficient $K_d$ are demonstrated once again to be very sensitive to small changes of elastics properties of the damaged material. Comparing to our previous numerical studies using a homogeneous model with a homogeneous EDZ (1), to match the experimental results of Zhang et al. (2), the required change of the attenuation coefficient in the EDZ has been effectively reduced. The multiple scattering effect through the cracks is clearly shown in the numerical tests, with which the necessary property changes can be significantly reduced. For future studies, numerical modeling in a very heterogeneous model will be conducted to study, with different crack distribution in the EDZ. The aim is to advance on a quantification of the damage level that can be applied in in-situ cases.

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References


